STIFFNESS OF THE HUMAN ARM

This web resource combines the passive properties of muscles with the neural feedback system of the short-loop (spinal) and long-loop (transcortical) reflexes and examine how the whole limb responds to a perturbation. One of us (RS), along with Ferdinando Mussa-Ivaldi and Emilio Bizzi performed an experiment where volunteers were asked to hold the handle of a robotic arm (Shadmehr and Mussa-Ivaldi, 1993). A sketch of the experimental setup is shown in Figure 1A. The measurements were taken at two different arm configurations, labeled “left” and “right” in this figure. For example, the hand was at the position labeled “right” in this figure and the robot displaced the hand from this position. The volunteers were instructed to close their eyes and “try not to intervene voluntarily” as the robot displaced their hand from this position.

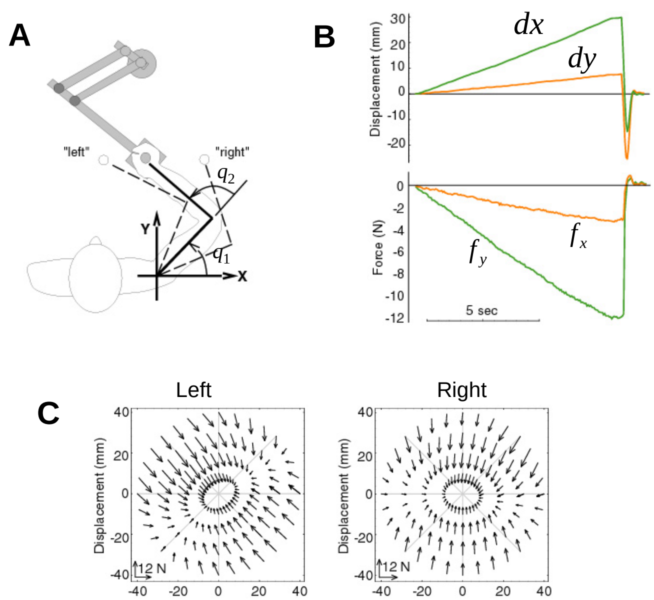

*Figure 1. **Measurement of arm stiffness at two different limb configurations.*** **A**. *Volunteers were asked to hold the handle of a robotic arm. Measurements were made at two different configuration of the arm: left and right positions.* **B**. *An example of force measured at the hand during displacement.* **C**. *Forces measured at the hand as a function of hand displacement. In the left subfigure, the arm is in the “left” position, while in the right subfigure the arm is in the “right” position. (From Shadmehr et al. 1993 (1993)).*

The robot’s handle housed a force transducer and it recorded the forces that the muscles produced as the hand was displaced from its equilibrium position. An example of one such displacement is shown in Figure 1B. Note that the displacement of the hand is a two-dimensional vector that has an x and a y component, and similarly the force measured at the hand has components f_x and f_y. As the hand is pulled farther from its equilibrium positions, the muscles produce a larger force, and the force measured at the hand increases. Eventually, the robot’s motors stop pulling at the hand and the arm snaps back to a position near the origin.

The forces that were measured as the hand was pulled at different directions are plotted in Figure 1C. The two subplots represent these forces when the arm was in the left and right configurations. We can immediately see that the pattern of forces changed significantly. When the arm is in the left configuration, pulling the hand along 135^o produces significant restoring forces. In contrast, when the arm is in the right configuration, the most significant restoring forces occur when the hand is pulled toward 45^o. Furthermore, note that the force vectors are often not pointing exactly toward the center (the origin). For example, with the hand in the left configuration, when the hand is pulled toward 270^o (straight down), the restoring forces push upward but also to the left.

The force pattern in Figure 1C allows us to visualize how passive properties of arm muscles combine with reflexes to produce an equilibrium position for the hand. Not all forces point to the center and some forces grow faster with displacement than others. Nevertheless, hand’s equilibrium position lies at the bottom of the “bowl” defined by these forces. The bowl is not round however, as its walls grow more steeply along certain directions. It is as if a spring is attached to the bottom of the bowl and its strength is different depending on which direction we pull it. Furthermore, the shape of the bowl seems to change as we move our arm from one configuration to another.

Muscles are producing these restoring forces. The relationship between the restoring forces and the hand’s displacement is somehow reflecting the stiffness of all the muscles that are being stretched in the arm. Whereas previously we were concerned with stiffness of a limb about a single joint, here we are concerned with the stiffness of the limb at the hand. How are these different kinds of stiffness related?

To answer this question, Ferdinando Mussa-Ivaldi, Neville Hogan, and Emilio Bizzi suggested that the restoring forces that were measured at the hand could be linearly approximated as a function of hand displacement (Mussa-Ivaldi et al., 1985). If we represent force at the hand and position of the hand as vectors:

$$\mathbf{f} \equiv \begin{bmatrix} f_x \\ f_y \end{bmatrix} \qquad \mathrm{x} \equiv \begin{bmatrix} x \\ y \end{bmatrix}$$

then stiffness, as measured at the hand, is the change in force with respect to change in position:

$$K_x \equiv \frac{d\mathbf{f}}{d\mathbf{x}} = \begin{bmatrix} \frac{df_x}{dx} & \frac{df_x}{dy} \\ \frac{df_y}{dx} & \frac{df_y}{dy} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \tag{1}$$

We put the subscript *x* under *K* here to remind ourselves that we are measuring stiffness in cartesian coordinates of the hand. Because force and displacement are two-dimensional vectors, stiffness will be a 2 x 2 matrix. That is, we will have to know how f_x changes with respect to displacements along *x* and *y*, and also know how f_y changes with respect to displacements along *x* and *y*. Our task is to estimate what the components of this stiffness matrix are.

To estimate stiffness, Mussa-Ivaldi and colleagues noted how much force they measured at the hand as they displaced it by a certain amount. They took *n* data points and their data set looked something like this:

$$\{d\mathbf{x}, d\mathbf{f}\}_1, \{d\mathbf{x}, d\mathbf{f}\}_2, \cdots \ \{d\mathbf{x}, d\mathbf{f}\}_n$$

Because displacements were from an arbitrary point where force at the hand was zero, let us drop the “*d*” and refer to the measurements as **x** and **f**. Our objective is to find a single matrix K_x such that over all the data points, the sum of the squared error between the estimated force and the actually measured force would be a minimum. To do this, we begin with a guess for K_x (some small random numbers will do). For our first data point, we multiply $\mathbf{x}_1$ by K_x and estimate the force:

$$\hat{\mathbf{f}}_1 = K_x \ \mathbf{x}_1$$

As K_x was merely a guess, we are bound to have an error in our estimate:

$$\varepsilon \equiv \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix} = \mathbf{f} - \hat{\mathbf{f}} = \begin{bmatrix} f_x - K_{11}x_1 - K_{12}y_1 \\ f_y - K_{21}x_1 - K_{22}y_1 \end{bmatrix}$$

We want to minimize the “square” of this error, but as this is a vector, we “square” it by multiplying it by its own transpose:

$$e \equiv \varepsilon^T \varepsilon = \left(f_x - K_{11}x_1 - K_{12}y_1 \right)^2 + \left(f_y - K_{21}x_1 - K_{22}y_1 \right)^2 \qquad (2)$$

We want to know how we should change our guess about the stiffness matrix so that this error is as small as possible. What we have done is to find a function in Eq. (2) that relates estimation error to our stiffness parameters. In a procedure called **gradient descent**, one takes the derivative of this error with respect to parameters of interest, that is, we find de/dK_{11} , de/dK_{12} , etc. If we change our parameters in a direction opposite to the one specified by the derivatives, error will be reduced. Therefore, we change our guess for each component of K_x by the following rule:

$$K_{ij} = K_{ij} - \eta \frac{de}{dK_{ij}} \qquad (3)$$

where η is a small constant. We now repeat this for the second data point, etc. By the time that Mussa-Ivaldi et al. got to the end of their *n* data points, and cycled through them a few times, they saw that K_x converged. Furthermore, it did a fair job of predicting the force that was recorded for any given displacement. To represent their result, they took the K_x that they had estimated for displacements of the hand and multiplied it by $d\mathbf{x}$, where $d\mathbf{x}$ was a vector that had a unit length and a direction that changed gradually from 0^o to 360^o. When the “circle” described by displacements $d\mathbf{x}$ was multiplied by the stiffness matrix K_x, it produced an ellipse. These ellipses are drawn for various configurations of the arm in Figure 2.

Compare the stiffness ellipses at the “left” and “right” configurations of the arm in this figure with the pattern of force in Figure 1. The long axis of each ellipse tells us the direction of hand displacement for which the restoring forces were maximum. The short axis of each ellipse tells us the direction of hand displacement for which the restoring forces were minimum. The arm is most stiff along a line that connects the hand to the shoulder joint. As the shoulder joint rotates, so does the direction of maximum stiffness.

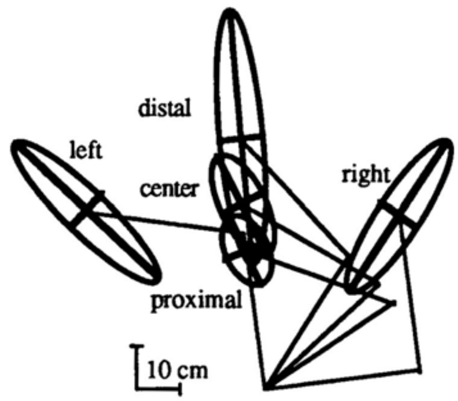

Figure 2. ***Representation of stiffness as measured at the hand as a function of arm configuration.*** *The 2x2 matrix that represents stiffness of the arm as measured at the hand, is plotted as an ellipse at each arm configuration. The major axis of each ellipse specifies the direction of hand displacement for which the limb is most stiff (at that position). The minor axis specifies the direction of displacement for which the limb is least stiff. (From Mussa-Ivaldi et al. 1985).*

Mussa-Ivaldi and colleagues observed that this basic pattern of stiffness was essentially the same across different volunteers (Mussa-Ivaldi et al., 1985). They wondered what it signified regarding stiffness of muscles and joints of the limb. To infer this, they transformed stiffness as measured in terms of displacements of the hand, called *end-point stiffness*, to a stiffness in terms of displacements in the joint angles, called *joint stiffness*. To do this, we need to transform forces **f** that are measured in terms of a cartesian coordinate system that represents hand position to torques τ. The torques are measured in terms of an angular coordinate system that specifies position of the joints, **q**. The Jacobian that describes the relationship between these two coordinate systems is defined as:

$$J(\mathbf{q}) = \frac{d\mathbf{x}}{d\mathbf{q}}$$

To compute the Jacobian, we write position of the endpoint (i.e., position of the hand) in terms of the joint angles:

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) \\ l_1 \sin(q_1) + l_2 \sin(q_1 + q_2) \end{bmatrix}$$

where l_1 and l_2 are the lengths of the upper and forearms, respectively. We next find the derivative of endpoint position with respect to the joint angle:

$$J(\mathbf{q}) = \frac{d\mathbf{x}}{d\mathbf{q}} = \begin{bmatrix} -l_1 \sin(q_1) - l_2 \sin(q_1 + q_2) & -l_2 \sin(q_1 + q_2) \\ l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2) \end{bmatrix}$$

Stiffness as measured in cartesian or joint coordinates is defined as:

$$K_x \equiv \frac{d\mathbf{f}}{d\mathbf{x}} \qquad K_j \equiv \frac{d\boldsymbol{\tau}}{d\mathbf{q}}$$

Using the principle of virtual work, we can relate force to torque using the Jacobian:

$$\boldsymbol{\tau} = J(\mathbf{q})^T \mathbf{f}$$

As joint stiffness is the derivative of the above function with respect to joint angular displacement, we have:

$$\begin{aligned} K_j &= \frac{d\left(J(\mathbf{q})^T \mathbf{f}\right)}{d\mathbf{q}} \\ &= \frac{d\left(J(\mathbf{q})^T\right)}{d\mathbf{q}} \mathbf{f} + J(\mathbf{q})^T \frac{d\mathbf{f}}{d\mathbf{q}} \end{aligned}$$

For small displacements of the limb, we can assume that the first term in the above sum is negligible, and so we have:

$$K_j \approx J(\mathbf{q})^T \frac{d\mathbf{f}}{d\mathbf{q}}$$

Expanding $d\mathbf{f}/d\mathbf{q}$ in terms of $d\mathbf{f}/d\mathbf{x}$ gives us an expression of joint stiffness in terms of end-point stiffness:

$$K_j \approx J(\mathbf{q})^T \frac{d\mathbf{f}}{d\mathbf{x}} \frac{d\mathbf{x}}{d\mathbf{q}} = J(\mathbf{q})^T K_x \, J(\mathbf{q}) \tag{4}$$

We now use this relationship to estimate joint stiffness for each configuration of the limb from the measured stiffness at the hand. The results are shown in Figure 3. In this figure, the joint stiffness matrix is used to transform a joint displacement that varies as a circle into an ellipse. There is a consistent shape to the ellipse and it does not vary very much as a function of configuration of the arm. Note how the major axis of the ellipse is approximately at 45^o. This means that the muscles produce a maximum restoring torque when both the elbow and shoulder joints are displaced in the same direction. That is, when both joints are either flexed or extended simultaneously, muscles produce a maximum restoring torque. When one joint is flexed but the other is extended, the restoring torque is minimum.

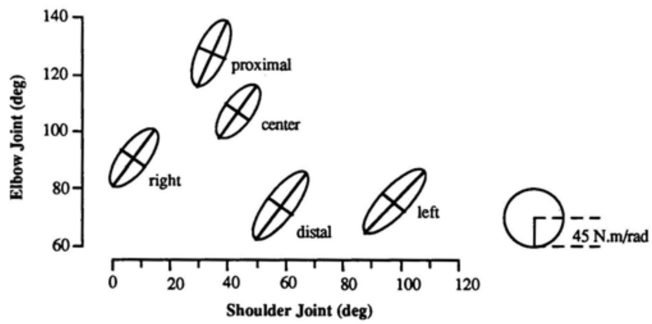

Figure 3. ***Joint stiffness as estimated from hand stiffness at various configurations of the arm.*** *(From Shadmehr (1993)).*

This pattern makes sense if you consider that we have both single joint and two-joint muscles in our arm. When both the elbow and shoulder joints are flexed, the stiffness of the single joint and double joints that extend these joints add to produce restoring torques on the joints. When one joint is flexed but the other is extended, the two-joint muscles may not change length, and therefore do not contribute to the restoring torques. As a result, the limb is least stiff for these displacements. The seemingly complex changes that we saw in endpoint stiffness were really the result of something rather simple at the level of the muscles.

Reference List

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