

$$\underline{df} \equiv \begin{bmatrix} df_x \\ df_y \end{bmatrix} \quad \underline{dx} \equiv \begin{bmatrix} dx_x \\ dx_y \end{bmatrix}$$

Data set: $\{\underline{dx}, \underline{df}\}_1, \{\underline{dx}, \underline{df}\}_2, \dots, \{\underline{dx}, \underline{df}\}_n$

Objective: find the best fit to $K_x = \frac{df}{dx}$ where $K_x = \begin{bmatrix} K_{x11} & K_{x12} \\ K_{x21} & K_{x22} \end{bmatrix}$

$$\underline{df} = K_x \underline{dx}$$

$$\text{error in fit for data set 1: } \varepsilon \equiv \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix} = \begin{bmatrix} df_x - K_{x11} dx_x - K_{x12} dx_y \\ df_y - K_{x21} dx_x - K_{x22} dx_y \end{bmatrix}$$

$$\text{cost function to minimize: } e \equiv \varepsilon^T \varepsilon = (df_x - K_{x11} dx_x - K_{x12} dx_y)^2 + (df_y - K_{x21} dx_x - K_{x22} dx_y)^2$$

$$\frac{de}{dK_{x11}} = -2(df_x - K_{x11} dx_x - K_{x12} dx_y) dx_x = -2\varepsilon_x dx_x$$

$$\text{Gradient descent: } (K_{x11})_{n+1} = (K_{x11})_n - \eta \frac{de}{dK_{x11}} = (K_{x11})_n + 2\eta \varepsilon_x dx_x$$

Arm's stiffness in joint coordinates

$$J(\underline{q}) = \frac{d\underline{x}}{d\underline{q}}$$

$$K_x = \frac{d\underline{f}}{d\underline{x}} \quad K_j = \frac{d\underline{\tau}}{d\underline{q}}$$

$$\underline{\tau} = J(\underline{q})^T \underline{f}$$

$$K_j = \frac{d(J(\underline{q})^T \underline{f})}{d\underline{q}} = \frac{d(J(\underline{q})^T)}{d\underline{q}} \underline{f} + J(\underline{q})^T \frac{d\underline{f}}{d\underline{q}}$$

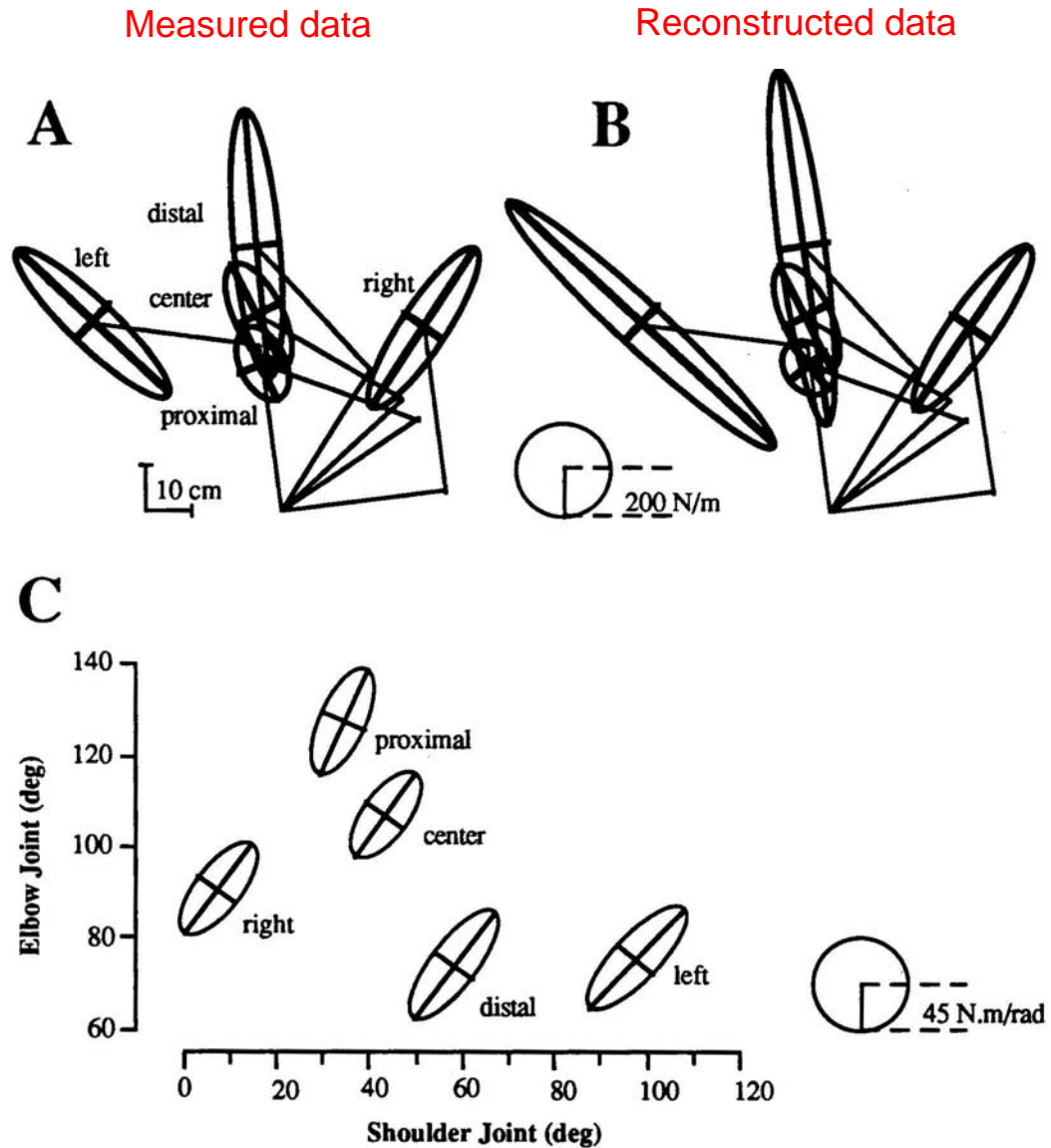
for small displacements, assume $\frac{d(J(\underline{q})^T)}{d\underline{q}} \approx 0$

$$K_j = J(\underline{q})^T \frac{d\underline{f}}{d\underline{q}} = J(\underline{q})^T \frac{d\underline{f}}{d\underline{x}} J(\underline{q})$$

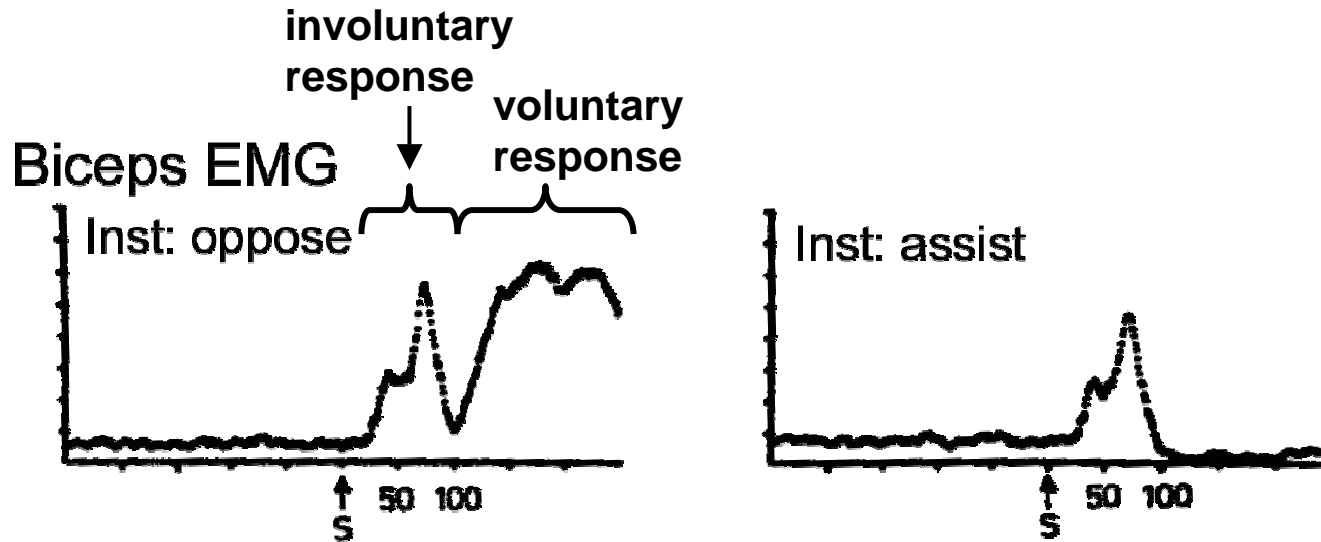
$$K_j = J(\underline{q})^T K_x J(\underline{q})$$

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There is remarkable regularity in the stiffness of the arm when it is expressed in joint coordinates. K_j is essentially constant and does not vary as a function of arm position. The reconstructed data is from the K_j measured at the center position.



Time delays in the short- and long-loop reflexes



Torque perturbations were imposed on the elbow of a subject at random times (S indicates the time of perturbation). On each trial, an LED instructed the subject to either oppose the torque or assist the torque.

Instruction dependent activity in the cerebellar dentate nucleus in response to a perturbation

