Smooth trajectories via a feedback controller: Bullock & Grossberg (1988)

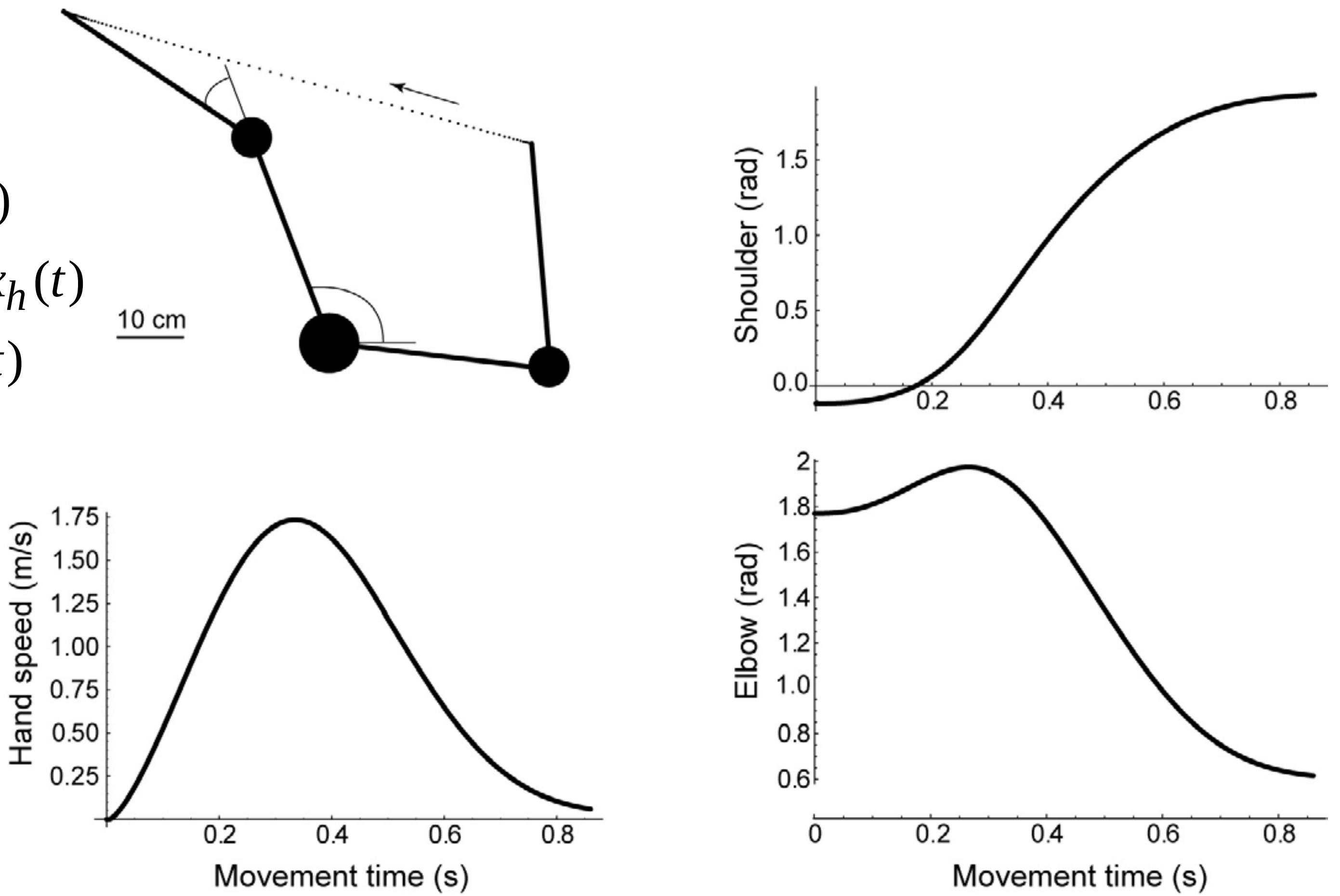
$$\Delta x_h(t) = \gamma(t) x_{dv}$$

$$\Delta\theta(t) = J(\theta)^{-1} \Delta x_h(t)$$

$$x_h(t+\Delta) = x_h(t) + \Delta x_h(t)$$

$$\theta(t+\Delta) = \theta(t) + \Delta\theta(t)$$

$$\gamma(t) = 0.1 t^{1/4}$$



Smooth trajectories via a feedback controller: Hoff & Arbib (1992)

The general solution to the minimum jerk functional was of the form $x^{(6)} = 0$

Normalized time $\tau = \dfrac{t - t_0}{D}$ $\quad D = t_f - t_0$

$$x(t) = a_0 + a_1\tau + a_2\tau^2 + a_3\tau^3 + a_4\tau^4 + a_5\tau^5$$

$$\dot{x}(t) = \frac{a_1}{D} + \frac{2a_2}{D}\tau + \frac{3a_3}{D}\tau^2 + \frac{4a_4}{D}\tau^3 + \frac{5a_5}{D}\tau^4$$

$$\ddot{x}(t) = \frac{2a_2}{D^2} + \frac{6a_3}{D^2}\tau + \frac{12a_4}{D^2}\tau^2 + \frac{20a_5}{D^2}\tau^3$$

Initial conditions:

$$x(t_0) = x_i \quad \dot{x}(t_0) = v_i \quad \ddot{x}(t_0) = p_i$$

$$a_0 = x_i \quad a_1 = Dv_i \quad a_2 = \frac{Dp_i}{2}$$

$$x(t_f) = x_f \quad \dot{x}(t_f) = 0 \quad \ddot{x}(t_f) = 0$$

$$a_3 = \frac{-3D^2}{2} p_i - 6Dv_i + 10(x_f - x_i)$$

$$a_4 = \frac{3D^2}{2} p_i + 8Dv_i - 15(x_f - x_i)$$

$$a_5 = -\frac{D^2}{2} p_i - 3Dv_i + 6(x_f - x_i)$$

$$q = \begin{bmatrix} x_i & v_i & p_i \end{bmatrix}^T \qquad D = t_f - t$$

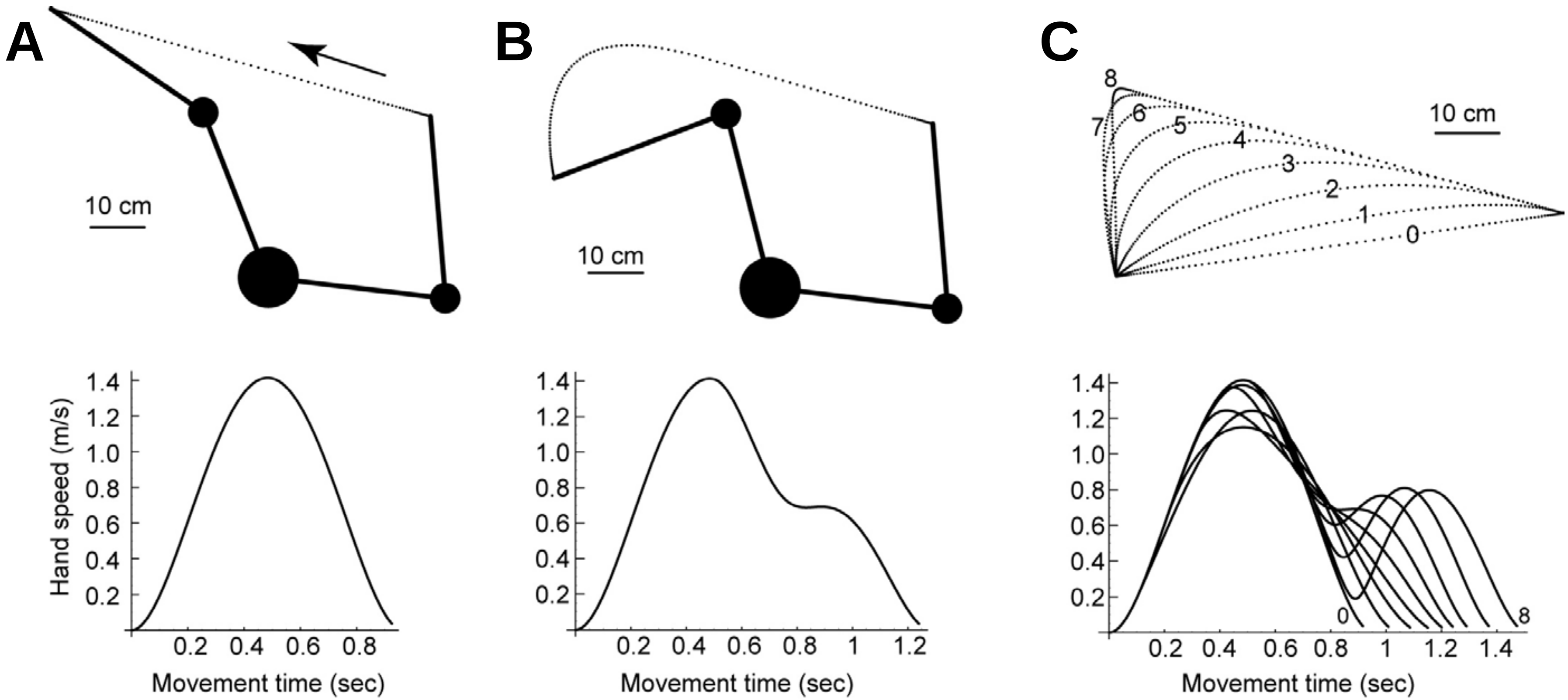
$$\dddot{x}(t) = \frac{6a_3}{D^3} + \frac{24a_4}{D^3}\tau + \frac{60a_5}{D^3}\tau^2$$

$$t = t_0 \quad \tau = 0$$

$$\dddot{x}(t_0) = \frac{6a_3}{D^3} = \frac{60}{D^3}(x_f - x(t_0)) - \frac{36}{D^2}\dot{x}(t_0) - \frac{9}{D}\ddot{x}(t_0)$$

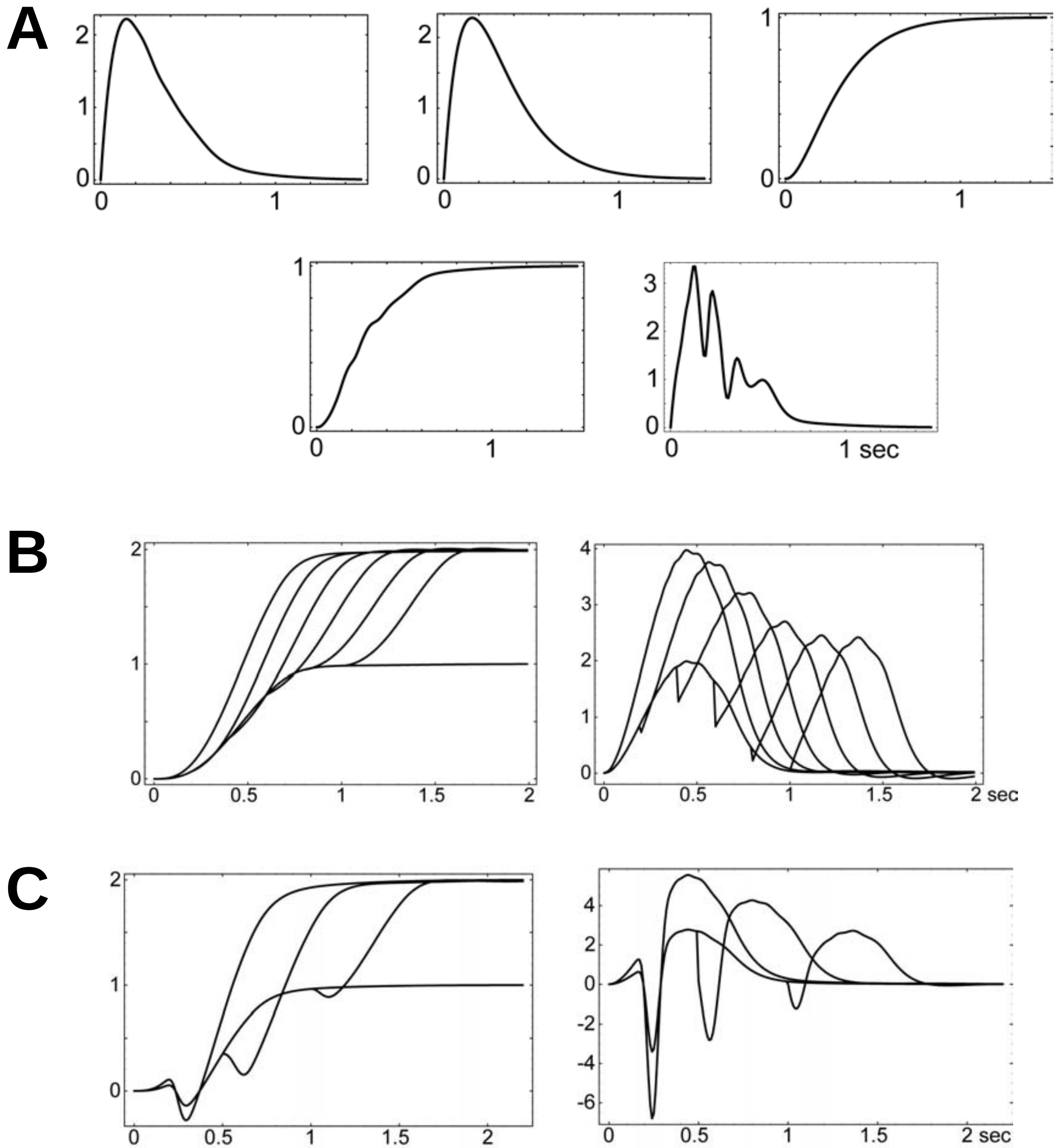
$$\dot{q} = Aq + Bx_f$$

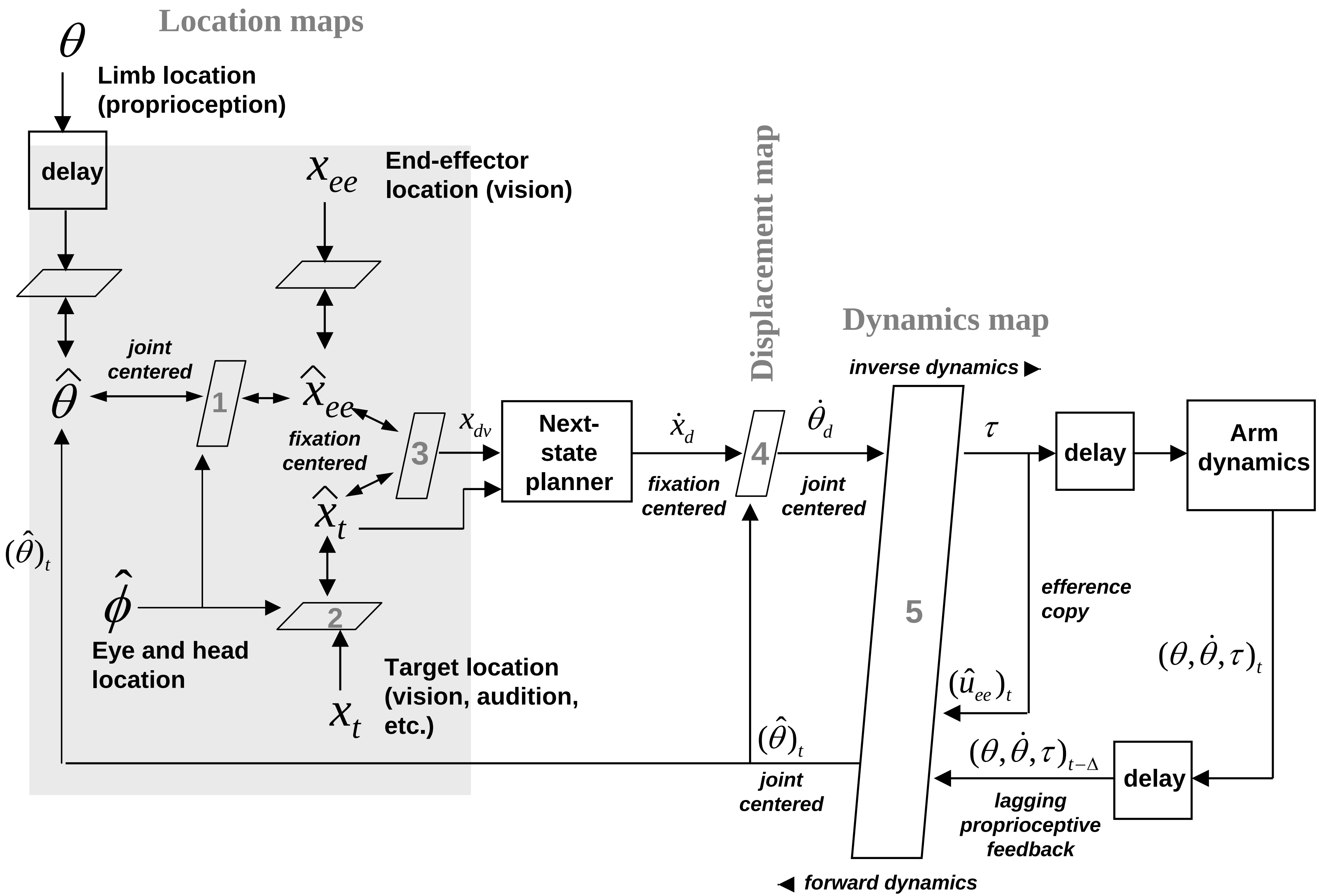
$$\dot{q} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{-60}{D^3} & -\frac{36}{D^2} & -\frac{9}{D} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{60}{D^3} \end{bmatrix} x_f$$



$$D = \left(60(x_t - \hat{x}_{ee})\right)^{1/3} r^{1/6}$$

A. D = 1.0 s. **B.** At t = 500 ms, the target jumped to a new location. At the time of target jump, time remaining (D) increased by 200 ms. **C**. Target jumped at t = 0 ms, t = 100 ms, …, t = 800 ms.


A
1 sec
B
2 sec
C
2 sec



Location maps
θ
Limb location (proprioception)
delay
x_{ee}
End-effector location (vision)
Displacement map
Dynamics map
inverse dynamics
$\hat{\theta}$
joint centered
1
$\hat{x}_{ee}$
fixation centered
3
x_{dv}
Next-state planner
$\dot{x}_d$
4
$\dot{\theta}_d$
5
τ
delay
Arm dynamics
fixation centered
joint centered
$\hat{x}_t$
$(\hat{\theta})_t$
$\hat{\phi}$
2
Eye and head location
Target location (vision, audition, etc.)
x_t
efference copy
$(\theta, \dot{\theta}, \tau)_t$
$(\hat{u}_{ee})_t$
$(\hat{\theta})_t$
$(\theta, \dot{\theta}, \tau)_{t-\Delta}$
delay
joint centered
lagging proprioceptive feedback
forward dynamics