Motor costs

Suppose we have 8 muscles that can each be activated by amount $u_i \geq 0$. Each muscle has a pulling direction specified by unit vector $\mathbf{p}_i$, such that the force $\mathbf{f}_i$ that is produces is:

$$\mathbf{f}_i = u_i \mathbf{p}_i$$

The resulting force produced by activation vector $\mathbf{u} = \left[u_1, \cdots, u_8\right]^T$ is:

$$\mathbf{f} = P\mathbf{u}$$

where $P = \left[\mathbf{p}_1, \cdots, \mathbf{p}_8\right]$. Suppose we are given a goal force $\mathbf{f}_g$, which is a unit vector pointing in some random direction. How should we activate the muscles? Let us describe a cost function that penalizes the difference between $\mathbf{f}_g$ and $\mathbf{f}$, as well as activations $\mathbf{u}$:

$$J = \left(\mathbf{f}_g - P\mathbf{u}\right)^T \left(\mathbf{f}_g - P\mathbf{u}\right) + \lambda \sum_i u_i^m$$

We want to see how the patterns of muscle activations change when we change the motor costs (i.e., the parameters λ and m).

Suppose that pulling direction of the muscles is specified as follows:

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Pa=[9, 49, 100, 145, 190, 230, 270, 340]; % deg
Pa=Pa/180*pi;
P=[cos(Pa); sin(Pa)];
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Further suppose that $\lambda = 0.5$ and $m = 1.1$. Using gradient descent with the constraint that the motor commands cannot be negative, find $\mathbf{u}^* = \left[u_1^*, \cdots, u_8^*\right]^T$ that minimizes the above cost for $\mathbf{f}_g = \left[1 \quad 0\right]^T$. Rotate $\mathbf{f}_g$ along a circle and recompute $\mathbf{u}^* = \left[u_1^*, \cdots, u_8^*\right]^T$. Plot your results in a form similar to Fig. 10.10 of Shadmehr and Mussa-Ivaldi (2012). Re-do this for $m = 2$ to see how increasing the motor costs affect the 'tuning function' of the muscles.