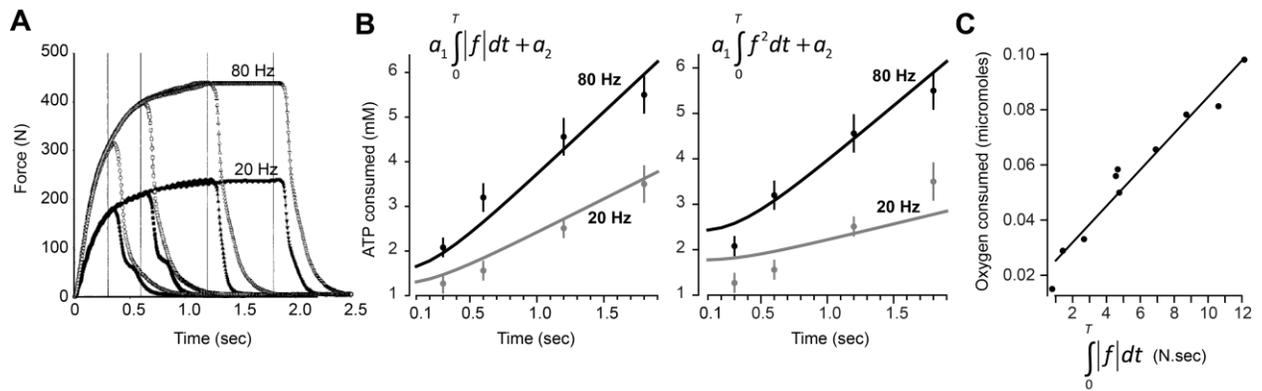


**Supplemental Information for:**

**A representation of effort in decision making and motor control**

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**Figure S1.** Related to Figure 3D-G. Energy consumed during isometric contraction grows linearly with the force-time integral. **A.** The measured force during electrical stimulation of the human gastrocnemius muscle at 20 or 80Hz (data from [S1]). The 20Hz stimulation produced a force that increased with a time constant of around 0.25sec, reaching a plateau of approximately 230N. The 80Hz stimulation produced a force that had a similar time constant, but reached a plateau of approximately 430N. **B.** Energy expended by the muscle as measured via consumption of ATP. The data points are from [S1]. In the left subplot, the curves depict a model where energetic costs grow linearly with the force-time integral. In the right subplot, the curves depict a model where energetic costs grow with the squared force-time integral. The fit with the force-time integral has about half of the errors as in the squared force-time integral. **C.** Oxygen consumed during electrical stimulation of a frog muscle plotted as a function of the force-time integral. Data from [S2].

## Supplemental Mathematical Derivations

### Speed of walking

Considerable research has approached the question of natural walking speed of various animals. Here, we briefly summarize this work, demonstrating that like reaching, energetics of human walking are convex in time, suggesting an optimum speed. However, this speed is slower than one that humans choose during locomotion. We suggest that the reason for this is that reward interacts with effort, making it worthwhile to be energetically inefficient.

The current view in walking research is that the energetic cost of locomotion defines the optimal speed of motion [S3]. For example, Ralston [S4] measured rate of energy expenditure in human walking and found that when people were not walking, the rate of energy expenditure per unit mass was  $\dot{e}_0 = a_0$ , and when they were walking  $\dot{e}_w = \dot{e}_0 + a_1 + bv^2$  per unit mass, where  $v$  is average speed of walking. Suppose that during an arbitrary period of time  $T$  a person spends some time  $T_w < T$  walking a distance  $d$ , and is otherwise not walking. During walking their average speed is  $d/T_w$ . The rate of energy spent during walking (per unit mass) becomes:

$$\dot{e}_w = a_0 + a_1 + \frac{bd^2}{T_w^2} \quad (S1)$$

If that individual has mass  $m$ , total energy spent during that  $T$  period is:

$$\begin{aligned} e_T &= a_0(T - T_w)m + (a_0 + a_1)mT_w + \frac{bmd^2}{T_w} \\ &= Ta_0m + a_1mT_w + \frac{bmd^2}{T_w} \end{aligned} \quad (S2)$$

To find the speed of walking that minimizes energy expenditure, we compute the derivative of the above expression with respect to  $T_w$ , and find the optimum walk duration  $T_w^*$ :

$$T_w^* = d \frac{\sqrt{b}}{\sqrt{a_1}} \quad (S3)$$

The optimum walking speed is:

$$v^* = \frac{d}{T_w^*} = \frac{\sqrt{a_1}}{\sqrt{b}} \quad (S4)$$

Therefore, the energetic cost of walking, similar to that of reaching, describes a convex function of time which exhibits a minimum. This means that in this energetics-only formulation of movement, there is an optimal speed and duration that minimizes the energetic cost of walking. Indeed, Hoyt and Taylor [S5] concluded that humans and other animals “select speed within a gait in a manner that minimizes energy consumption”.

However, this energetics-only framework has a number of limitations. As Srinivasan [S3] has pointed out, the optimum speed predicted by Eq. (S4) is considerably slower than the speed that people naturally choose to walk. The reason for this, our theory suggests, is that reward discounts effort, making it worthwhile to expend energy to acquire a rewarding goal (Figure 1). One caveat is that the optimum walking speed depends on how energetic cost is represented. A common approach in the locomotion literature [S4] is to represent it as the total energetic cost to walk a unit distance, referred to as the cost of transport. The optimum speed predicted by minimizing the total cost of transport provides

a reasonable match to the natural speeds observed. However, an argument against optimization of total cost of transport is that it relies on the unrealistic assumption that one will always be walking and nothing else.

Ultimately, regardless of how energetic cost is calculated, an energetics-only framework does not consider a role for reward. As a result, this framework cannot account for the observation that people and other animals move faster toward stimuli that promise greater reward [S6-S10]. It cannot account for the observation that more impulsive people may move faster than less impulsive people [S11]. It cannot account for the observation that movement speed declines with increasing inter-trial intervals [S12]. It cannot account for the observation that people in different cities walk at different speeds [S13].

Because we found that energetics of walking and reaching were both convex functions of time, our results regarding effects of reward, mass, distance, etc. on vigor of reaching predict similar effects on vigor of walking.

In summary, our framework extends the existing approach in the field of walking research in a critical way: using energetics of action as a measure of effort, we define a utility in which there is interplay between reward, effort, and time. We arrive at a theory that not only describes where one should walk (toward the choice that offers the greatest utility), but also how fast one should walk (via a speed that maximizes the utility). As a result, the theory provides an account of the observation that when there is greater reward at stake, one should walk faster. When time is more valuable, one should walk faster. If mass increases, one should walk slower.

### **Combining effort and reward additively vs. multiplicatively**

We suggested that effort discounted reward additively. This is in contrast to earlier work where it was assumed that effort discounts reward multiplicatively [S14-S16]. Let us compare these two approaches. In the case of multiplicative interaction between reward and effort, we have the following representation for utility:

$$J = \frac{\alpha}{1 + \gamma T} U(T) \quad (S5)$$

In the above formulation increasing reward increases the utility of the action, but has no effect on the optimal movement duration (because the effect of reward is to scale the utility function, which has no effect on the value of time that maximizes the utility). Therefore, such a formulation is inconsistent with the observation that reward makes movements faster. That is, any utility in which reward is multiplied by a function of effort will fail to predict sensitivity of movement vigor to reward. This is in contrast to experimental evidence [S10]. In contrast, an additive interaction between reward and effort correctly predicts that increased reward increases movement vigor.

### **Hyperbolic vs. exponential temporal discounting**

We suggested that like reward, effort is discounted hyperbolically. In reinforcement learning [S17], future rewards are discounted exponentially, largely due to mathematical convenience. Exponential discounting has also been suggested in models of human decision making [S18]. Hyperbolic discounting, however, is more consistent with decision-making data in humans [S19] and monkeys [S20]. In our expression for movement utility, we opted to discount reward and effort as a hyperbolic function of time. The reason for this is that exponential discounting makes the incorrect prediction that changes in inter-trial intervals should have no effect on movement vigor. To illustrate this, suppose that time discounts utility exponentially. For an arbitrary effort  $e(T)$ , we have:

$$J = (\alpha + e(T)) \exp\left(-\frac{T+q}{\tau}\right) \quad (S6)$$

If we take the derivative of the above equation with respect to  $T$ , we find the following expression:

$$\frac{dJ}{dT} = \left[ \frac{de(T)}{dT} - \frac{\alpha + e(T)}{\tau} \right] \exp\left(-\frac{T+q}{\tau}\right) \quad (S7)$$

To find the optimum duration, we set the above expression equal to zero, and find that the inter-trial delay  $q$  has no consequence on the optimum duration. Thus, in the case of exponential discounting we find that vigor is independent of inter-trial delay, something that is inconsistent with experimental evidence [S12].

We also considered the possibility that reward and effort were discounted exponentially in time in our analysis of the choices that birds made in walking vs. flying, and the choices people made in isometric force production. For the walking vs. flying data set, we found that exponential discounting of reward and effort provided as good a fit to the data as hyperbolic discounting ( $R=0.99$ ,  $p<10^{-8}$ ). For isometric force production (Figure 3), we found that exponential discounting did a better job than no discounting at all, but still underperformed hyperbolic discounting ( $R=0.84$ ,  $p<10^{-7}$ ).

In Figure 3D, the indifference curves dip downward, and then move slightly upward with increasing durations. This is unexpected because it suggests that the utility of producing a force for a given duration is equal to producing a larger force for a larger duration. Intriguingly, exponential discounting can account for the curious dip observed in the indifference curves, although the overall fit to the data was still worse than hyperbolic discounting. The dip could also be explained by a utility in which time was discounted hyperbolically, where discounting of time is due to a power function of time:

$$U = \frac{\alpha - a_1 FT - a_2}{1 + T^\gamma} \quad (S8)$$

While the above utility could account for the curious dip, its overall fit to the data ( $R=0.65$ ,  $p<10^{-3}$ ) was not as good as the utility where effort is hyperbolically discounted by time.

### Integral of squared motor commands vs. energetic cost

Assuming that temporal discounting of effort is hyperbolic, is there a fundamental difference between representing effort via energetic cost as compared to sum of squared forces? We will consider this question in this section and show that sum of squared forces presents a cost that is convex in time but has no global minimum. In contrast, energetics of both walking and reaching are convex in time but also have a global minimum. The implications of this result are presented.

Let us describe a utility that represents effort via integral of squared forces. How does this utility depend on duration of movement? Suppose a movement is made in period  $T$ , along trajectory  $q(t)$ . We compute the integral of the squared forces for that movement, represented by  $s$ . Now suppose that the system makes the movement in period  $T/a$ . What is integral of the squared forces produced in this movement? We note that:

$$\begin{aligned} \bar{q}(t) &= q(r(t)) \\ r(t) &= at \end{aligned} \quad (S9)$$

The velocity of the movement  $\bar{q}(t)$  is related to the movement  $q(t)$  by the following:

$$\frac{d\bar{q}(t)}{dt} = \frac{dq(r)}{dt} = \frac{dq(r)}{dr} \dot{r} = q'(r)\dot{r} = aq'(r) \quad (S10)$$

For acceleration we have:

$$\begin{aligned} \frac{d^2\bar{q}(t)}{dt^2} &= \frac{d(q'(r)\dot{r})}{dt} = \frac{dq'(r)}{dt} \dot{r} + q'(t)\ddot{r} = \frac{dq'(r)}{dr} \dot{r}^2 + q'(t)\ddot{r} = q''(r)\dot{r}^2 + q'(t)\ddot{r} \\ &= a^2 q''(r) \end{aligned} \quad (S11)$$

For a typical system that has dynamics described by inertial and coriolis/centripetal forces, we have the following relationship between forces and motion:

$$\tau(t) = I(\mathbf{q}(t))\ddot{\mathbf{q}}(t) + C(\mathbf{q}(t))\dot{\mathbf{q}}^2(t) \quad (\text{S12})$$

For the sped-up trajectory, we have:

$$\begin{aligned} \bar{\tau}(t) &= I(\bar{\mathbf{q}}(t))\frac{d^2\bar{\mathbf{q}}(t)}{dt^2} + C(\bar{\mathbf{q}}(t))\left(\frac{d\bar{\mathbf{q}}(t)}{dt}\right)^2 \\ &= I(\mathbf{q}(r))a^2\mathbf{q}''(r) + C(\mathbf{q}(r))a^2\mathbf{q}'(r) \\ &= a^2\tau(r) \end{aligned} \quad (\text{S13})$$

The above result implies that when we scale time to go from one trajectory that has duration  $T$  to another that has duration  $T/a$ , the forces scale by  $a^2$ . Let us now compare the integral of the squared forces in the two trajectories. We indicate this integral with the symbol  $i(T)$  for one trajectory, and  $i(T/a)$  for the other trajectory.

$$\begin{aligned} i(T) &= \int_0^T \tau^2(t)dt = s \\ i(T/a) &= \int_0^{T/a} \bar{\tau}^2(t)dt = \int_0^T a^4\tau^2(r)\frac{dr}{a} = a^3s \end{aligned} \quad (\text{S14})$$

These two equations imply that there exists a function  $i(T)$  such that:

$$\frac{i(T)}{i(T/a)} = \frac{1}{a^3} \quad (\text{S15})$$

A function that satisfies the above condition is:

$$i(T) = \frac{c}{T^3} \quad (\text{S16})$$

This result implies that the sum of squared forces required for moving an inertial system along an arbitrary trajectory will decrease as the third power of the duration of movement. These forces scale approximately linearly with mass. As a result, we conclude that if our measure of effort is the sum of squared forces, then the utility of a constant amplitude movement as a function of duration  $T$  and mass  $m$  is:

$$J = \frac{\alpha - cm^2/T^3}{1 + \gamma T} \quad (\text{S17})$$

In comparison, if our measure of effort is the energetic cost of that movement, then the utility is:

$$J = \frac{\alpha - amT - bm/T^2}{1 + \gamma T} \quad (\text{S18})$$

We find that a fundamental difference between representations of effort via sum of squared forces vs. energetic cost is that the latter is a convex function of time with a minimum, whereas the former is not. The minimum arises from the bias term,  $am$ , in Eq. (1) that represents an energetic cost that grows with time, penalizing movements with longer durations. The bias term is not present if effort is sum of squared forces or even sum of forces. It also cannot be explained by the cost of supporting the arm against gravity, as the arm was supported throughout the movement. Interestingly, a similar bias is present in walking, and also leads to energetics that are convex in time with a minimum [S4].

What is the implication of a movement duration that minimizes energetic cost? Consider a thought experiment in which one is given the option of doing nothing (and receiving no reward), vs. performing a movement for  $T = 10$  seconds to receive reward  $\alpha$ . If we set  $\alpha$  large enough, then both utilities produce a positive value, which implies that one should choose to perform the movement and receive the reward. Now suppose that we increase the required movement duration. If the utility of these choices is based on squared forces, as  $T \rightarrow \infty$ ,  $J \rightarrow 0$ , but always remains positive. This means that according to the squared forces model, if we accept to perform a short duration movement for some reward, we must also accept it despite the requirement of extremely long movement durations. This unreasonable prediction arises because representation of effort via sum of squared forces does not exhibit a minimum as a function of time.

However, if the utility of the choice is based on energetic cost, then as  $T \rightarrow \infty$ , then  $J \rightarrow -\frac{am}{1+\gamma T}$ . This means that regardless of reward, the utility of doing an action for a very long period is always less than zero. As a result, while we will accept to perform the movement for a short period of time in exchange for the reward, as the movement duration becomes longer, we will reject the offer of reward and opt to do nothing. This reasonable prediction arises because the energetic cost has a global minimum.

### Effective mass of the hand as a function of movement direction

In Figure 2 we considered data from experiments in which people reached to various directions. To compute the utility of each movement, we needed to estimate the effective mass of the arm as it moved in each direction. Here, we show how to estimate this effective mass.

We begin with the inertia of the arm, which for the planar configuration is a 2x2 position-dependent matrix  $I(\theta)$ , where  $\theta = [\theta_s \quad \theta_e]^T$ , representing angular position of the shoulder and elbow joints. At rest, inertia represents the relationship between a vector of joint accelerations and the resulting torque:

$$\tau = I(\theta)\ddot{\theta} \quad (S19)$$

We are interested in computing the mass matrix  $M(\theta)$  which represents the relationship between the acceleration vector  $\ddot{x}$  and the force vector as measured at the hand at rest:

$$f = M(\theta)\ddot{x} \quad (S20)$$

We use the Jacobian matrix:

$$\Lambda = \frac{dx}{d\theta} \quad (S21)$$

and the principle of virtual work to relate force to torque, and acceleration in joint coordinates to hand coordinates:

$$\begin{aligned} \tau &= \Lambda^T f \\ \dot{x} &= \Lambda \dot{\theta} \\ \ddot{x} &= \frac{d\Lambda}{d\theta} \dot{\theta} \dot{\theta} + \Lambda \ddot{\theta} \end{aligned} \quad (S22)$$

Using the above equalities, we can write the relationship between hand acceleration and force:

$$f = (\Lambda^{-1})^T I(\theta) \Lambda^{-1} \ddot{x} - (\Lambda^{-1})^T I(\theta) \Lambda^{-1} \frac{d\Lambda}{d\theta} \dot{\theta} \dot{\theta} \quad (S23)$$

When the hand is at rest, the velocity-dependent term on the right side of the equation is zero, providing the following relationship between acceleration and force:

$$f = (\Lambda^{-1})^T l(\theta) \Lambda^{-1} \ddot{x} \quad (S24)$$

As a result, we can define the mass matrix at the hand as:

$$M(\theta) = (\Lambda^{-1})^T l(\theta) \Lambda^{-1} \quad (S25)$$

In the case of the planar arm that we are considering, the mass matrix  $M(\theta)$  is a 2x2, describing the relationship between accelerations and forces at the hand. To compute the effective mass  $m(\theta)$  we apply an acceleration of length unity in a given direction and compute the length of the resulting force vector.

### Energetic cost of isometric force production

We considered data from an experiment in which energy consumption was estimated via the change in ATP concentration (or alternatively, oxygen uptake) during the electrical stimulation of an isometric muscle (Figure S1A). We modeled the actual forces produced by the muscle, computed its integral, and then fitted the following equation to the measured data, with the results shown in Figure S1B (left panel):

$$e = a_1 \int_0^T |f(t)| dt + a_2 \quad (S26)$$

We found that this model produced a mean-squared error that was about half as large as those produced if we had assumed that energy was related to the integral of the squared force:

$$e(f(t)) = a_1 \int_0^T f^2(t) dt + a_2 \quad (S27)$$

The results are shown in the right subplot of Figure S1B. Other experimental data [S2] provided further evidence for the conjecture that the energetic cost associated with isometric force production is a linear function of the force-time integral (Figure S1C).

Let us consider the iso-utility curves in Figure 3D and ask what would happen if effort was the energetic cost of force production, but not discounted in time. This means that the utility of producing a constant force  $F$  for duration  $T$  is:

$$J = \frac{\alpha}{1 + \gamma T} - a_1 FT - a_2 \quad (S28)$$

To find iso-utility curves, we solve the equality  $J(F_1, T_1) = J(F_2, T_2)$  and obtain the following expression:

$$F_2 = \frac{a_1 F_1 T_1}{a_1 T_2} - \frac{\alpha}{a_1 T_2 (1 - \gamma T_1)} + \frac{\alpha}{a_1 T_2 (1 - \gamma T_2)} \quad (S29)$$

This equation tells us that if effort is not temporally discounted, then the iso-utility curves will asymptote to zero with increasing duration  $T_2$ . Therefore, undiscounted energetic cost, like the time-integral of force (Figure 3G), will lead to iso-utility curves that go to zero, which is inconsistent with the experimental data.

## Supplemental Tables

**Table S1. Simulation Parameters and Goodness of Fit for Figures 1 and 2**

		Parameters										Goodness of Fit
Simulation	Fig	$a$ (J/s/kg)	$b$	$i$	$j$	$\alpha$ (kJ)	$\gamma$	$m$ (kg)	$d$ (m)	$\theta$ (sec)	$k$	
Movement Utility	1B	15	77	1.1	3	0.5	1	2	0.1	0	n/a	n/a
Effect of reward	1C	15	77	1.1	3	0.5 1.0	1	2	0.1	0	n/a	n/a
Effect of mass	1D	15	77	1.1	3	0.5	1	2 3	0.1	0	n/a	n/a
Effect of impulsivity	1E	15	77	1.1	3	0.5	1 2	2	0.1	0	n/a	n/a
Effect of inter-trial delay	1F	15	77	1.1	3	0.5	1	2	0.1	0.5 1	n/a	n/a
Effect of mass on movement duration (Gordon et al. 1994 )	2B	15	77	1.1	3	0.31	1	$m(\theta)$	0.1	0	n/a	R = 0.83, $p < 10^{-34}$
Effect of mass on movement direction (Wang and Dounskaia 2012)	2C	15	77	1.1	3	0.1	1	$m(\theta)$	0.15	0	n/a	Right: R=0.67, $p < 10^{-8}$ Left: R=0.68, $p < 10^{-9}$
Effect of mass on movement direction (Cos et al. 2011)	2H 2I	15	77	1.1	3	0.1	1	$m(\theta)$	0.11 0.075- 0.145	0	28	T1/T2: R=0.96, $p=0.009$ T3/T4: R=0.96, $p=0.008$

**Table S2. Simulation Parameters and Goodness of Fit for Figure 3A-3C**

		Parameters								Goodness of Fit
Simulation	Fig	$\dot{e}_w$ (J/s)	$\dot{e}_f$ (J/s)	$\dot{e}_p$ (J/s)	$t_w$ (sec)	$t_f$ (sec)	$t_p$ (sec)	$\alpha$ (kJ)	$\gamma$	
Walk/Run Indifference Points: Temporal discounting of reward and effort	3A	2 J/s	31.7	1.09	0.6	1.1	1.25	1.3	0.03	R=0.99, p<0.0001, SSE=74.35
Walk/Run Indifference Points: No temporal discounting	3B	2 J/s	31.7	1.09	0.6	1.1	1.25	1.3	0	R=0.99, p<0.0001, SSE=757.84
Walk/Run Indifference Points: Temporal discounting of reward only	3C	2 J/s	31.7	1.09	0.6	1.1	1.25	1.3	0.007	R=0.99, p<0.0001, SSE=109.28

**Table S3. Simulation Parameters and Goodness of Fit for Figure 3E-3G**

		Parameters				Goodness of Fit
Simulation	Fig	$\alpha$	$a_1$	$a_2$	$\gamma$	
Temporal discounting of reward and effort: Force-Time Integral	3E	1	1	1	25	R=0.92, p<10 <sup>-11</sup>
No temporal discounting: Force <sup>2</sup> -Time Integral	3F	1	1	n/a	n/a	R=0.67, p<10 <sup>-4</sup>
No temporal discounting: Force-Time Integral	3G	1	1	n/a	0.001	R=0.62, p<10 <sup>-3</sup>

## Supplemental Experimental Procedures

### EXPERIMENT: Energetic cost of reaching

We measured rate of metabolic energy expenditure via expired gas analysis as subjects made reaching movement of different distances and durations. Fifteen healthy adults (mean  $\pm$  SD age  $24.2 \pm 4.4$  yrs, 8 females, 7 males) participated in this study. All subjects were right-handed and recruited from the University of Colorado Boulder student body. The University of Colorado Institutional Review Board approved the study protocol and all subjects gave informed consent.

Seated subjects grasped the handle of a robotic arm (Interactive Motion Technologies, Shoulder-Elbow Robot 2) to move a circular cursor from a home circle to a target circle at five pre-determined reaching speeds. The cursor, home circle, and target circle were displayed on a vertically mounted computer screen at the subject's eye-level. The five speeds are referred to as Very Slow, Slow, Medium, Fast, and Very Fast. We tested three reach distances of 10, 20, and 30cm. On odd numbered trials, reaches started  $\sim 15$  cm in front of the chest area with the arm in a flexed position. On even numbered trials, the reach started at the previous target location with the arm in an extended position and involved flexing the arm back towards the center target. Subjects wore bilateral shoulder straps and a lap belt to limit torso movement. A cradle attached to the robot handle supported the right forearm against gravity and restricted wrist movement.

A training bar that moved with a velocity that corresponded to the minimum jerk trajectory was used to illustrate the desired reaching speed during a familiarization period at the beginning of each reaching block. Additionally, the target turned gray if the reach was too slow, green if the reach was too fast, and "exploded" as a flashing yellow ring if the reach was within  $\pm 50$ ms of the desired movement duration. A pleasant auditory tone was also used to signal that the subjects successfully hit the target within the desired time window. After reaching the target, subjects had 800ms to settle in the center ring of the home circle before the next target circle was displayed. Thus, the inter-trial time was fixed at 800ms for all speeds at each reach distance.

We measured metabolic rate using expired gas analysis (ParvoMedics, TrueOne 2400). Subjects wore a nose clip and breathed in and out of a mouthpiece during data collection. We sampled the rates of oxygen consumption ( $\dot{V}_{O_2}$ ) and carbon dioxide production ( $\dot{V}_{CO_2}$ ) at 5 second intervals as subjects made reaching movements at the desired speeds. Data collections occurred early in the morning, after subjects had fasted overnight. We calibrated the system prior to each data collection using certified gas mixtures and with a range of flow rates using a 3 liter calibration syringe. All metabolic data were corrected with standard temperature and pressure, dry (STPD).

Subjects performed five 5-minute reaching blocks at each of the five fixed speeds. The speeds for these five reaching blocks were randomized for each subject. Each 5-minute reaching block began with 20 practice trials during which no energetic data was recorded. After the practice trials, subjects placed the clip on their nose, inserted the mouthpiece, and breathed for  $\sim 1$  minute while sitting quietly. After this 1-minute breathing period, subjects performed N number of reaches, where N was chosen to last  $\sim 5$  minutes. Thus, all subjects performed the same number of reaches for a given reaching speed and reach distance. In between reaching blocks, subjects rested for at least 5 minutes during which no energetic data was recorded. If subjects were naïve to reaching with the robotic arm, we asked them to come in for a brief  $\sim 15$  minute familiarization session the day prior to the data collection. The familiarization session involved short reaching blocks of 50 trials at relatively slow and fast speeds until the subject appeared to be comfortable with the robotic arm and the task. Only the trials performed during the last 3 minutes of each reaching block, corresponding to the steady-state metabolic data, were used in the calculations.

We only analyzed metabolic data with respiratory exchange ratio,  $RER = \dot{V}_{CO_2} / \dot{V}_{O_2}$ , values less than 1.0 and generally below 0.85, suggesting that oxidative metabolism was primarily involved [S21]. Normal resting RER values range from 0.74 to 0.87, partly depending on diet and other factors [S22, S23].

We calculated the average gross metabolic rate to perform the task in terms of Joules per second using the Brockway equation [S24]. As we were interested only in the cost of moving the arm, we calculated the net metabolic rate by subtracting the bias representing the cost of sitting quietly with the hand resting in the cradle. To obtain a movement's net metabolic cost in units of Joules, we multiplied the measured net metabolic rate in J/s for each movement duration and distance, by trial duration. Trial duration is the time between the start of consecutive movements, which represents the sum of the movement duration, the inter-trial time, and reaction time. The net metabolic rate for a movement of a fixed distance and duration was then calculated as the net metabolic cost of the movement divided by the movement duration. Note that  $\dot{e}_r$  represents the average not the instantaneous metabolic rate over the course of a movement. We next parameterized the relation between a movement's net energetic rate and the duration and distance of the movement. We fit metabolic rate to Eq. (1), to determine the free parameters  $a$ ,  $b$ ,  $i$ , and  $j$ . The distance  $d$  was known. The effective mass of the arm and robot was estimated as  $m = 2$  kg, based on an inertial model of the arm using standard anthropometric values (see section on Mass of the Arm and Movement Vigor for more details regarding the arm model).

## SIMULATIONS

Goodness of fit for each model was determined by calculating the correlation coefficient and the sum of squared errors, SSE. Parameter values and goodness of fit for each simulation are provided in Supplemental Tables S1-S3. Simulation code is available upon request.

### Mass of the arm and movement vigor

We tested the predictions of the theory in conditions where the mass of the limb was varied via the direction of the reach (Figure 2). We considered an inertial model of the human arm that was composed of two segments, with the following properties:

$$d_1 = 0.33 \quad d_2 = 0.43 \quad \text{meters}$$

$$m_1 = 1.93 \quad m_2 = 1.52 \quad \text{kg}$$

$$\lambda_1 = \frac{d_1}{2} \quad \lambda_2 = \frac{2d_2}{3} \quad \text{meters}$$

$$I_1 = 0.014 \quad I_2 = 0.019 \quad \text{kg m}^2$$

In the above expressions,  $d_i$  is length of each segment,  $m$  is mass,  $\lambda_i$  is length from point of rotation of the segment to its center of mass, and  $I$  is the inertial of the segment, with the subscript 1 referring to the upper arm, and subscript 2 referring to the forearm and hand. To predict what the movement duration and velocity should be for each direction  $\theta$ , we first computed the effective mass along that direction  $m(\theta)$  by computing the length of the vector that resulted when an acceleration of  $1 \text{ m/s}^2$  in the direction of movement was multiplied by the mass matrix  $M$ . The result was scalar value function  $m(\theta)$ , which was then used to compute the predicted duration for a reach in that direction, with an amplitude of 10cm, that is,  $d = 0.1$  (as shown in Figure 2B). We then computed the peak velocity of the resulting movement using a minimum-jerk trajectory.

To compute effort of each movement, we used the energetic costs that we had measured in Figure 1A, and set  $\gamma = 1$ , with the only free parameter  $\alpha$ . We found that a value that provided a good fit to the data of Gordon et al. [S25] was  $\alpha = 310$ .

### Mass of the arm and choice of the movement

We next tested the predictions of the theory by considering the choices that people made when they were free to choose their own movement direction. In this experiment, the right and left arms were placed in a given configuration and the subjects were asked to make an out-and-back reaching movement to a circle of 15cm radius, but to a direction of their choice [S26]. The resulting probability distribution of the directions that they chose is shown in Figure 2C (gray region). To see whether our theory could account for the data, we fit the data, with only one free parameter,  $\alpha$ , keeping all other parameters unchanged from the simulations shown in Figure 2B. We found that a value that provided a good fit to the data was  $\alpha = 100$ .

We first computed the effective mass for the left and right arms for the out-and-back movement by using the mass at the start point and each possible turn-around point about a circle of 15cm radius. We then used this effective mass to predict the duration of each 30cm movement, and then to compute the utility of that movement. For each possible movement direction we computed its utility, and then computed the ratio of this utility to the sum of utilities across all movement directions, producing the following probability:

$$\Pr(\theta_i) = \frac{J(\theta_i)}{\sum_i J(\theta_i)} \quad (\text{S30})$$

The results are shown with the black curve in Figure 2C, with the effective mass distribution for an out-and-back movement plotted in Figure 2D. In Figure 2E, the sum of utilities for quadrants 1 and 3 is compared to the fractions of trials that the subjects chose to reach to those quadrants.

Our formulation of utility function was further tested by considering the choices that people made when they were given the option of reaching to one of two possible targets (Figure 2F). The idea was that for each target, the effective mass of the movement described the utility for that movement, and the difference in the utilities associated with the two targets should describe the probability of choosing one target over another. We kept the parameters that we had found in Figure 2C unchanged. This produced a utility function with nothing to fit. To compute the effective mass for the reach to a given target, we computed the effective mass at the start and end points and averaged the two. To compute the probability of choosing a target, we used a logistic function in which the probability was a function of the difference in the utility of each target. Target T1 had a utility that was 16% larger than the utility of target T2. When the two targets were equally distant from the start point, the subjects chose target T1 on around 80% of the trials. To model the choice of targets as a function of movement distance, we used a logistic function:

$$\Pr(T1) = \left[ 1 + \exp \left( k \left( \frac{1}{2} - \frac{J_{T1}}{J_{T1} + J_{T2}} \right) \right) \right]^{-1} \quad (\text{S31})$$

Here,  $k$  is a free parameter representing noise in the decision making process and was fixed to the same value in all simulations. Figure 2H illustrates the fit of the function to the data for the probability of choosing target T1 over target T2, and target T3 over target T4. As the distance to target T1 and T3 increases, the preference shifts to target T2 and T4. In the logistic function, the only free parameter was  $k$ , which we found to be 28 for the data in Figure 2I.

### To walk or to fly

One of our main ideas is that representation of effort may depend on temporally discounted energetic cost of action. To test this idea, we considered an experiment in which both the caloric values of reward and the energetic cost of performing the action were objectively known.

Bautista et al. [S27] trained starlings to choose between walking a short distance or flying a long distance to acquire a piece of food. The caloric content of this reward was known,  $\alpha = 1.3 \times 10^3$ , as was the energetic rate associated with walking, perching, and flying:  $\dot{e}_w = 2$ ,  $\dot{e}_p = 1.09$ , and  $\dot{e}_f = 31.7$ .

Time spent in each act was also known:  $t_w = 0.6$ ,  $t_p = 1.25$ , and  $t_f = 1.1$  sec. According to our theory, the utility function takes the following form:

$$J = \frac{\alpha - e}{1 + \gamma T} \quad (\text{S32})$$

Here,  $T$  is total travel time,  $e$  is energetic cost of the movement, and  $\gamma$  is a temporal discounting factor. The variable  $T$  represents the time the animal spends performing three different activities: moving, perching, and handling the food. The movement time  $t_w$  is the time the birds spend walking in a one-way trip (or time spent flying, represented as  $t_f$ ). In addition to walking or flying, the animals spent time perching in between walking or flying one-way trips ( $t_p$ ), and there is additional time spent handling the reward ( $t_h$ ) before they consumed it. If the animal chose to walk  $n_w$  times to acquire reward, the travel time is:

$$T_w(n_w) = 2n_w(t_w + t_p) + t_h \quad (\text{S33})$$

Bautista et al. [S27] estimated the energetic rate during perching  $\dot{e}_p$  from previous recordings and assumed that the energetic rate of handling  $\dot{e}_h$  was the same. Thus the total energetic cost for making  $n_w$  walking trips is:

$$e_w(n_w) = 2n_w(\dot{e}_w t_w + \dot{e}_p t_p) + \dot{e}_p t_h \quad (\text{S34})$$

Combining the above equations, we find the utility for the choice of taking  $n_w$  walking trips:

$$J_w(n_w) = \frac{\alpha - (2n_w(\dot{e}_w t_w + \dot{e}_p t_p) + \dot{e}_p t_h)}{1 + \gamma(2n_w(t_w + t_p) + t_h)} \quad (\text{S35})$$

We can similarly define the utility for flying,  $J_f$ .

The indifference point is found by setting  $J_w(n_w) = J_f(n_f)$  and solving for  $n_w$ . The only free parameter is  $\gamma$ . We found that changing  $\gamma$  had a monotonic effect on the indifference curve: as Figure 3A illustrates, small  $\gamma$  (patient animal) led to a preference for walking, whereas large  $\gamma$  (impulsive animal) led to a preference for flying. The data was best fit for  $\gamma = 0.022$ .

If neither reward nor effort is temporally discounted, then there are no free parameters in the utility function. The number of walks is proportional to the number of flights, where the proportion is determined by the ratio of the energetic cost of flying to walking (Figure 3B).

Finally, if reward is temporally discounted but not effort, then once again there is only one free parameter,  $\gamma$ . However, in this case changing  $\gamma$  has a non-monotonic effect on the indifference curve. When  $\gamma$  is very small (patient animal,  $\gamma = 0.0001$ ), there is a preference for walking (Figure 3C). When  $\gamma$  is very large (impulsive animal,  $\gamma = 1$ ), there is once again a preference for walking (Figure 3C). The closest that we can come to the measured data is with  $\gamma = 0.007$ , which provides a poor fit.

Indeed, if we assume that effort is not temporally discounted, the mean-squared error between the model and data is an order of magnitude larger than if effort is temporally discounted.

### **Sensitivity of results to parameter values**

We chose to represent utility as a sum of effort and reward. A number of earlier approaches have used a multiplicative approach, where effort multiplicatively discounts reward [S14-S16]. An additive formulation is a better fit to the data because a multiplicative interaction cannot account for the observation that increased reward results in increased movement vigor [S10, S28].

We chose to discount the utility via a hyperbolic function of time. Earlier works have suggested an exponential discounting [S18]. Exponential discounting cannot account for the observation that vigor declines with increased inter-trial interval [S12]. Hyperbolic discounting performs as well or better, compared to exponential discounting in the choices birds made in walking vs. flying and the choices people made in isometric force production.

We considered the possibility that only reward but not effort may be temporally discounted. Such a scenario produced a model that had the same number of parameters as in the utility where both effort and reward were temporally discounted. However, we found that temporal discounting of reward and effort always produced a better fit to the measured data in choices that birds made in walking vs. flying, and choices that people made in isometric force production.

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