The network consists of three input layers and an intermediate layer. The three input layers – an eye-centered layer, an eye position layer and a head-centered layer – are also output layers; the final estimates of the network are read from these layers after relaxation. The three input layers consist of three topographic layers of N units indexed by their position, \(i(\text{or} j,k)\), where \(i(\text{or} j,k) = 1\ldots N\). Similarly, the intermediate layer is a topographic 2D map of \(N \times N\) units indexed by their position \(l, m\), where \(l = 1\ldots N\) and \(m = 1\ldots N\).

1. Connection weights
The input layers are symmetrically interconnected with the intermediate layer (hidden layer), and the corresponding matrices of connection weights are denoted by \(W^e\), \(W^c\), \(W^a\) for, respectively, the eye-centered, eye position and head-centered layers. The connection strengths between unit \(i(\text{or} j,k)\) in each input layer and unit \((l,m)\) in the intermediate layer are given by
The variable $\sigma_w$ represents lateral spread: unit $i$ is strongly connected if $|i - l|/N \leq \sigma_w/2\pi$. Note that with these connection matrices, unit $(l,m)$ in the intermediate layer is most strongly interconnected with unit $i=l$ in the eye-centered layer, $j=m$ in the eye position layer and $k=l+m$ in the head-centered layer. Unit $(l,m)$ is connected more weakly to neighboring units in each layers, with the spatial extent of these connections is controlled by $\sigma_w$.

2. Network initialization

$R_{ri}(t)$, $R_{ej}(t)$, and $R_{ak}(t)$ are denoted as the activity of unit $i (j,k)$ in the eye-centered, eye-position and head-centered layer at time $t$.

For eye-centered position, the probability distribution for the initial activity, denoted $R_{ri}(0)$, is given by

$$P(R_{ri}(0)|x_r) = \frac{f_i(x_r)^{R_{ri}(0)} \cdot e^{-f_i(x_r)}}{\Gamma(R_{ri}(0))},$$

$$f_i(x_r) = C_i (K \exp \left[ \frac{\cos[2\pi / N(i - l)] - 1}{\sigma^2_w} \right] + \nu)$$

The expressions for $P(R_{ej}(0)|x_e)$ and $P(R_{ak}(0)|x_a)$ are identical to $P(R_{ri}(0)|x_r)$, except that $r$ is replaced by $e$ or $a$. The expressions for $f(x_a)$ and $f(x_e)$ are identical to $f_i(x_r)$ except that $r$ is replaced by $a$ or $e$.

The activity in the intermediate layer, $A_{lm}(0)$, is initialized to 0: $A_{lm}(0) = 0$ for all $l,m$.

3. Recurrent network evolution

The evolution of the activities in the recurrent network is described by a set of coupled nonlinear equations. Denoting $A_{lm}(t)$ as the activity of unit $(l,m)$ in the intermediate layer at time $t$, the evolution equations are written

$$L_{lm}(t) = \sum_i W_{ril}^e R_{ri}(t) + \sum_j W_{jlm}^e R_{ej}(t) + \sum_k W_{kml}^a R_{ak}(t)$$

$$A_{lm}(t + 1) = \frac{L_{lm}(t)^2}{S + \mu \sum_{l,m} L_{lm}(t)^2}$$
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\[ R_s(t+1) = \frac{\left( \sum_{l} \sum_{m} W_{ilm}^e A_{lm}(t+1) \right)^2}{S + \mu \sum_{l} \sum_{m} W_{ilm}^e A_{lm}(t+1)} \]

\[ R_j(t+1) = \frac{\left( \sum_{l} \sum_{m} W_{jlm}^e A_{lm}(t+1) \right)^2}{S + \mu \sum_{l} \sum_{m} W_{jlm}^e A_{lm}(t+1)} \]

\[ R_{al}(t+1) = \frac{\left( \sum_{l} \sum_{m} W_{ilm}^a A_{lm}(t+1) \right)^2}{S + \mu \sum_{l} \sum_{m} W_{ilm}^a A_{lm}(t+1)} \]

where \( L_{ilm}(t) \) represents a linear pooling of activities from the three input layers. The activation functions \( A_{lm} \) or \( R_{sj}, R_{ej}, R_{ak} \) implement a quadratic nonlinearity coupled with a divisive normalization.

4. Parameters used in the simulation

\[ K = 20 \text{ Hz}, \nu = 1 \text{ Hz}, \sigma = 0.40 \text{ radians}, K_w = 1, \mu = 0.002 \text{ s} \text{ and } S = 0.1 \text{ Hz}. \]

\[ \sigma_w = 0.37 \text{ radians}, C_r = C_e = C_a = 1 \text{ s}. \]