Transforming a desired trajectory into a control policy

How do we build a next-state planner that produces an arbitrary trajectory? Perhaps the simplest way is to first record that trajectory, and then transform that trajectory into a set of parameters for the next-state planner. Here we will use the approach described by Stefan Schaal and his colleagues {Ijspeert et al. 2002/d} to produce a next-state planner that generates minimum jerk trajectories and zigzag trajectories. The parameters describe the control policy for our next-state planner.

The next-state planner has two inputs, x_t and $\hat{x}_{ee}$ (target position and estimated end-effector position), and one output, $\dot{x}_d$ (desired change in end-effector position). It has four internal states, labeled z, v, x, $\tilde{x}$, and a few time constants a_z, b_z, etc. A set of differential equations describe the dynamics of this system:

$$\dot{z} = a_z\left(b_z(x_t - x_d) - z\right) \tag{1}$$

$$\dot{v} = a_v\left(b_v(x_t - x_d) - v\right) \tag{2}$$

$$\dot{x} = v\left(1 + a_{px}(\hat{x}_{ee} - x_d)\right)^{-1} \tag{3}$$

$$\tilde{x} = (x - x_o)/(x_t - x_o) \tag{4}$$

$$\dot{x}_d = z + \frac{\sum_{i=1}^{n} w_i g_i(\tilde{x})}{\sum_{i=1}^{n} g_i(\tilde{x})} v + a_{py}(\hat{x}_{ee} - x_d) \tag{5}$$

In this formulation, x_o represents end-effector position at the start of the movement, and $\tilde{x}$ is a measure of the fraction of the distance progressed to the target. The distance $\tilde{x}$ is encoded via a set of nonlinear basis function g_i:

$$g_i = \exp\left(-\frac{1}{2\sigma_i^2}(\tilde{x} - c_i)^2\right) \tag{6}$$

The parameters of this system are the weights w_i and the time constants a_z, b_z, etc.

Suppose that we wish the system to generate a plan $\dot{x}_d(t)$ such that the end-effector moves from its initial position $x_o = 0$ to target position $x_t = 1$ in $t_d = 1$ second. Further, suppose that we wish the desired movement to be minimum jerk. Therefore, we have:

$$x_d(t) = x_o + (x_t - x_o)\left(10(t/t_d)^3 - 15(t/t_d)^4 + 6(t/t_d)^5\right)$$
$$\dot{x}_d(t) = (x_t - x_o)\left(30(t/t_d)^2 - 60(t/t_d)^3 + 30(t/t_d)^4\right) \quad (7)$$

(These functions are defined for the period $t = 0 \rightarrow t_d$ and maintain their final value for times greater than t_d.) Because at this stage we are concerned with finding the parameters that produce our plan, we are going to assume that in Eqs. (3) and (5) that $\hat{x}_{ee}(t) = x_d(t)$, i.e., our planned movement is precisely executed. Therefore, Eqs. (3) and (5) become

$$\dot{x} = v \quad (8)$$

$$\dot{x}_d = z + \frac{\sum_{i=1}^{n} w_i g_i(\tilde{x})}{\sum_{i=1}^{n} g_i(\tilde{x})} v \quad (9)$$

The fact that we want the movement to end at around 1 second dictates the time constants of the differential equations. For example, the larger we set a_z and a_v, the faster the system will want to converge to the goal. In this example, we will set the time constants to be:

$$a_z = a_v = 12$$
$$b_z = b_v = a_z / 4$$

We plug in $\dot{x}_d(t)$ in Eq. (2), calculate $\dot{v}$, and using rectangular integration iterate this equation to compute $v(t)$. A similar procedure is done for Eq. (1) to compute $z(t)$. Next, we compute $x(t)$ from Eq. (8) and use the result to compute $\tilde{x}(t)$ in Eq. (4). The basis functions g_i encode this space. In this example, we used 25 bases with centers equally disturbed over the range of 0-1 with $\sigma_i = 1/25$. Therefore, all the elements of Eq. (9) are now known except for the weights w_i. The weights appear linearly in the equation and can be found using linear algebra.

To see how well the parameters produce our intended trajectory, we run Eqs. (1-5) and compare the planned trajectory $\dot{x}_d(t)$ with our intended one in Eq. (7).